Study and Analysis of the Capacity of Ad Hoc Wireless Networks using Mobility and Delay

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Abstract: with a cellular architecture of \( m \) base stations, it is investigated that each wireless node can be realized with a throughput that By employing a practical and simple routing policy to study and analyse the capacity using mobility and delay of an ad-hoc wireless network and consisting of \( n \) static wireless nodes overlaid scales sub linearly or linearly with \( m \). The analysis shows that one requires a large deployment cost in order to achieve a \( \Theta(1) \) capacity. Existing research works also indicate that for pure mobile ad hoc networks, a capacity of \( \Theta(1) \) can be achieved by exploiting the mobility of the nodes, at the expense of very high end-to-end delay. This larger delay, nevertheless, stems from the assumption of global mobility, where nodes move around the entire network. By leveraging a more practical and restricted mobility model, it is analysed the capacity using mobility and delay ad-hoc wireless network design with \( n \) mobile nodes and \( m \) base stations, termed as mobile hybrid wireless network. This results show that each node can be realized with a capacity of \( \Theta(1) \), while keeping the average end-to-end delay smaller by a factor of \( m \) than the pure mobile ad hoc networks.

Keywords: component; Capacity, Mobility, Delay, Mobile Hybrid Wireless Networks, Static Hybrid Wireless Network

I. INTRODUCTION

It has been recently recognized that augmenting base stations to pure ad hoc wireless networks [2], [3]-[4],[11] commonly referred to as hybrid wireless networks, can indeed render larger benefits in terms of capacity, mobility and delay. One can envisage these base stations as a means to carry all the long distance transmissions from a source node, through the wired network, to its intended destination. The reduced number of hops denotes smaller packet delay for nodes. The capacity of static hybrid wireless networks has been significantly accelerated the throughput capacity under two different routing strategies [5]. Specifically in \( k \)-nearest cell routing strategy, it is shown that if \( m \) grows asymptotically slower than \( \sqrt{n} \), the maximum per node capacity scales as \( \Theta \left( \frac{1}{\sqrt{n \log n/m^2}} \right) \). If \( m \) grows asymptotically faster than \( \sqrt{n} \), the maximum per node throughput capacity scales as \( \Theta \left( \frac{W/m}{n} \right) \) which in turn offers a better throughput gain dependent on \( m \). Importantly, in the region \( m = \Theta(\sqrt{n}) \), one can attain only less than log \( k \)-fold benefit on capacity as the number of base stations are increased from \( m \) to \( km \). In [9], it is investigated capacity figures. By studying these existing efforts in depth, three major issues in the existing capacity analysis are identified:

ISSUE I

The capacity analysis fails to account for the fact that adding base stations to the pure ad hoc wireless networks plays a critical role in decreasing the number of hops between each S-D pair and thereby, the amount of traffic flowing through each relaying node. One may observe that failure to include this aspect in capacity analysis can lead to inaccurate results.

ISSUE II

Each node in the network is assumed to communicate with its base station using a one-hop wireless uplink. First, this implies that those power constrained wireless nodes has to transmit at higher power levels to reach their associated base stations. However, such assumptions are energy-inefficient, especially when wireless nodes are configured to transmit at \( < 100 \) as opposed to the base stations that can transmit at \( 20W \) – \( 60W \) [1]. Second, it also implies that nodes require a clear Line-of-Sight (LoS) between the node and the base station for effective communication.

ISSUE III

In [5], to formulate the final capacity expression, the parameter \( n \) in \( W \left( \frac{n}{\log n} \right) \) (capacity of pure ad hoc network) is simply substituted with \( \frac{m}{\log n} \) which the number of nodes is communicating in ad hoc mode. Such substitutions indirectly imply that nodes are restricted to communicate only with a limited number of nearby neighbours. However, such a model cannot guarantee the network connectivity when certain base stations break down.

In [9], these existing issues are resolved by proposing a simple and practical routing policy referred to as same cell routing policy. In this policy, a source node routes its packets to the destination using multiple hops only if both the source and its destination are located in the same cell. Otherwise, the packets are initially transmitted using multiple hops to the base station which eventually forwards all the packets to the destination as in a cellular network. Given \( W \) bits/sec as the total bandwidth, for a SHWN with nodes and \( m \) base stations, it is identified the following two regimes:

(I) When \( m = O \left( \frac{n}{\log n} \right) \). In this regime, a per node throughput capacity of \( \Theta \left( \frac{m}{n \log n} \right) \) is achieved. It also
follows that if the numbers of base stations are increased from \( m \) to \( km \), one can actually obtain a gain of \( \sqrt{k} \) on capacity as opposed to [5], which only provides a less than \( \log k \) fold increase on capacity. The average delay is bounded by 
\[
\Theta \left( \frac{n}{\sqrt{m \log n}} \right)
\]  
(2) When \( m = \Omega \left( \frac{n}{\log n} \right) \). In this regime, a per node throughput capacity of \( \Theta \left( W \frac{m}{n} \right) \) is achieved. The average delay is bounded by \( \Theta(1) \).

### A. Motivation and Contribution

The throughput capacity of \( \Theta(W) \) can be obtained only with a large number of base stations, i.e., when \( m = \Theta(n) \), which in turn implies a large deployment cost. A constant throughput scaling of \( \Theta(W) \) can be obtained by leveraging the mobility characteristics of a node in an ad hoc network [3]-[4], [11]. It is assumed that global mobility where the nodes can move around the whole network, therefore address the following question: Can a scheme design that can realize a \( \Theta(W) \) throughput capacity by exploiting the mobility characteristics of the nodes, while possibly keeping the end-to-end delay smaller?

This question is addressed by leveraging the mobility characteristics of the nodes in a hybrid wireless network under same cell routing policy. Particularly for a mobile hybrid wireless network (MHWN), when source and its destination is located in the same cell (or different cell), the routing policy requires nodes to send its packets by moving towards the destination (or base station) instead of using multi-hop transmissions. Each node is only allowed to move within a limited region (i.e., in a cell) rather than the entire region. Such a restriction will allow obtaining high capacity and reduced delay in a cost-effective manner for hybrid wireless networks. This analysis identifies the following two regimes for MHWN with \( n \) nodes and \( m \) base stations:

1. When \( m = (n) \). In this regime, a per node throughput capacity of \( \Theta(W) \) is achieved. The average delay is bounded \( \Theta \left( \frac{n \log n}{m} \right) \).

2. When \( m = \Omega(n) \). In this regime, a per node throughput capacity of \( \Theta(W) \) is achieved. The average delay is bounded by \( \Theta(1) \). From the results it follows that, though \( m = O(n) \), a per node capacity of \( \Theta(W) \) capacity can be still realized by keeping the delay (i.e., \( n \log m \)) smaller than pure mobile ad hoc networks (i.e., \( n \log n \)) [4].

### NETWORK MODEL

#### A. Hybrid Wireless Network Model

This paper considers a mobile hybrid wireless network (MHWN), consisting of \( n \) mobile wireless nodes, overlaid with a cellular architecture of \( m \) base stations on a planar torus of unit area. A hybrid wireless network consists of two layers, an ad hoc layer and a cellular layer. The ad hoc layer assumes that \( n \) wireless nodes are uniformly and independently (randomly) distributed on the surface of a unit area torus and each node leverages NCAICN-2013, PRMIR, Badnera same transmission power to communicate with its neighboring nodes or base stations. It considers that each node is a source of exactly one flow and a destination node for at most \( O(1) \) flows. It is assumed that a stand-alone wireless network at the ad hoc layer. As a result, even in the absence of any base stations, nodes can still engage in communication with its chosen destinations; this assumption solves ISSUE III in [5]. The cellular layer regularly deploys \( m \) base stations, at the top of ad hoc layer, in such a manner that it tessellates the plane into equal-sized squares of area \( \frac{1}{m} \). In the jargon of cellular networks, each square calls as a cell and at the centre of each cell, a base station is placed.

Unlike wireless nodes, base stations neither serve as data sources nor as data receivers. Instead, they serve as relays to forward the traffic for wireless nodes in the ad hoc layer. Moreover, the base stations are also assumed to be connected to each other with a very high bandwidth network so that there are no bottlenecks associated with the base stations. In contrast to wireless nodes, it is also assumed that there are no power constraints for the base stations. Finally, to ensure that the mutual interference between base stations remains below a threshold, it is assumed that adjacent cells employ a frequency reuse policy similar to a cellular network [1], [9].

#### B. Routing Policy for Mobile Hybrid Wireless Networks

This paper considers a simple and practical routing policy called as same cell routing policy for mobile hybrid wireless networks. Under this policy, if the source-destination pair lies in the same cell, a source node initially forwards its data to one or more relay nodes, which in turn moves around within the cell until it reaches the intended destination and transmits the data. This mode of data transmission is referred as mobile mode. The nodes are allowed to move within a cell rather than the entire network. This in turn reduces the time required for sending the data to the final destination. If the source and destination are located in two different cells, source node forwards the data to its nearest base station using relays (i.e., using mobile mode) and the base station as in a cellular network forwards the data through the wired network to the destination node. And, this is referred to as hybrid model of data transmission as hybrid mode, a combination of mobile mode and cellular transmission. If the destination/ base station is within the transmission range of a source node, it transmits data directly to it without relying on the relay nodes. Therefore, it clearly turns out that, independent of the location of source and its destination, in a MHWN each packet is constrained to take at most two hops i.e., \( \Theta(1) \).

#### Data Rate:
Each node is assumed to transmit at a maximum data rate of $W$ bits per second over a common wireless channel of bandwidth. This wireless channel is partitioned into three subchannels each of bandwidth, $W_A$ for mobile ad hoc transmissions in a MHWN, $W_D$ for uplink transmissions to the base stations and $W_B$ for downlink transmissions from the base stations, respectively. Since the amount of traffic in the uplink and downlink channels are the same, so $W_D = W_A$. As a result, the sum of the transmission rates due to radio transmissions as well as base station related transmissions can be expressed as $W = W_A + 2W_B$.

C. Definitions

1) Throughput

A per-node throughput of $\Lambda(n, m)$ bits per second, for a hybrid wireless network of $n$ nodes and $m$ base stations is said to be achievable, if every node can transmit data to its chosen destination at a rate of $\Lambda(n, m)$ bits per second. The throughput capacity of the hybrid wireless network with $n$ nodes and $m$ base stations are expressed by $\Lambda(n, m) = \Lambda_D(n, m) + \Lambda_B(n, m)$, where $\Lambda_D(n, m)$ and $\Lambda_B(n, m)$ denote the throughput capacity contributed by the mobile ad-hoc transmissions in a MHWN and the base station relative transmissions (i.e. uplink and downlink) respectively. Since there are $n$ source-destination pairs, the network capacity can be defined to be $n \Lambda(n, m)$.

2) Average Packet Delay

The delay of a packet is the time it takes for the packet to reach the destination after it leaves the source. Thus, the per packet delay is the sum of the times a packet spends at each relay node. The average packet delay of a hybrid network $(n, m)$ is then obtained by averaging over all transmitted packets in the network due to the ad hoc transmissions.

II. MHWN: CAPACITY AND DELAY ANALYSIS

This section establishes the upper and lower bounds on the throughput capacity and delay of MHWN under same cell routing policy. The related theorems are stated as follows.

**Theorem 1**: For a MHWN with $n$ nodes and $m$ base stations, the throughput capacity $\Lambda(n, m)$ furnished to each node under the same cell routing policy is:

$$\Lambda(n, m) = \Theta(W_D) + \Theta\left(\frac{m}{n} W_B\right),$$

where $\Lambda_D(n, m) = W_D$ and $\Lambda_B(n, m) = m/n W_B$.

**Theorem 2**: For a MHWN with $n$ nodes and $m$ base stations, the average delay $(n, m)$ of each packet under the same cell are:

$$D(n, m) = \frac{n \log n}{m}$$

As the mobile ad hoc and base station relative transmissions are carried in two different sub channels, the final capacity will be $\Lambda_D(n, m) + \Lambda_B(n, m)$.

A. Lower bound: Capacity and Delay for mobile ad hoc transmissions

In this section, a routing scheme for MHWN is constructed and is shown that it achieves a constant throughout of $W_A$. The ad hoc layer of MHWN and deduce the unit area region by sub cells are of area $(n) = 1/n$. As shown in Fig. 1, at the top of this ad hoc layer, a virtual layer is laid out i.e., cellular layer, formed by $m$ cells, each of size $\sqrt{n} \times \sqrt{n}$. To be more precise, such a construction will result in each cell of area $\frac{1}{m}$ to consist of $\frac{1}{ma(n)}$ subcells, of area $a(n)$ each. The transmission range of each node is chosen to be $2\sqrt{2}$ times the side length of the sub cells, i.e., $2\sqrt{2n^{-1/2}}$.

Once the network is constructed, a scheme to route the data to the final destination is constructed. For this purpose, a straight line is drawn, that passes through some sub cells, connecting each S-D pair. In a MHWN under same cell routing policy, if a S-D line lies completely inside (or outside) a cell, the packets are transmitted from source to destination (or base station) in mobile mode. For transmission, the time slot at which each node can transmit its packets is determined. A sub cell X is said to interfere with another sub cell Y, if there is a sender in sub cell X which is within a distance $(2 + \Delta)n(n)$ of some sender in sub cell Y. The parameter $\Delta$ defines the size of the exclusion region and hence, $\Delta > 0$.

**Lemma 1**: The number of sub cells that interfere with any given sub cell is bounded by a constant $c_3 = (2 + \Delta)^2$, i.e. independent of $n, m$ and $a(n)$.

From Lemma 1, it turns out that each sub cell in the network becomes active once in every $1 + c_3$ slots. In an active sub cell, each required node that is randomly chosen to transmit its packet to another node lying in its sub cell. Thus, depending on whether the node is source or relay, each slot is further divided into two sub-slots A and B. In sub-slot A, each node acting as a source sends its packet directly to destination/base station if its present in the same/adjacent sub cell. Otherwise, it sends its packet to another randomly chosen node present in its/neighbor sub cell, which acts as a relay. In sub-slot B, each node acting as a relay sends its packet directly to the destination/base station if its present in the same/adjacent sub cell. It is to be noted that since each packet is directly sent to its destination/base station or relayed at most once, the total number of routes passing through a sub cell in a cell, $E[Z] + E[Z] = \Theta(1)$.

**Lemma 2**: Let $a(n) = \frac{1}{n}$ be the area of the sub cell and let $p_{sub}$ be the probability that any subcell has at least two nodes.

Then, $p_{sub} = 1 - \frac{2}{e}$.
Proof: An arbitrary sub cell is chosen. The probability \( p_{\text{sub}} \) that there are at least two nodes in a sub cell is given by:

\[
p_{\text{sub}} = 1 - \left( \frac{1}{n} \right)^n - \left( \frac{1}{n} \right)^{n-1} \\
\geq 1 - e^{-1} - e^{-\left( \frac{n}{n-1} \right)} = 1 - 2e^{-1}
\]

where the second inequality is based on the fact that \((1 - x) \leq e^{-x}\).

The Lemma mainly says that the above routing scheme can be successfully executed by nodes with a probability no less than \( p_{\text{sub}} \).

1) Throughput Capacity

The lower bound on the throughput capacity contributed by the mobile ad hoc transmissions, \( \Lambdaa \), is based on the above routing scheme. The proof is essentially based on Lemma 1 and Lemma 2. First, from Lemma 1 and the fact that \( E[Z] + E[Z] = \Theta(1) \), it follows that each sub cell can be active for a constant fraction of time period of length \( \frac{1}{1+\epsilon_3} \) seconds. Further, Lemma 2 suggests that on average an active sub cell can have transmissions for at least \( \frac{1}{2(1+\epsilon_3)} \) seconds during a slot of length \( \frac{1}{1+\epsilon_3} \) with a probability no less than \( p_{\text{sub}} \). Each node can successfully transmit for \( \frac{1}{2(1+\epsilon_3)} \) fraction of time at the rate of \( W \) bps with a probability no smaller than \( p_{\text{sub}} \). Thus, \( \Lambdaa \) is:

\[
\Lambdaa \approx \frac{\Omega}{\frac{1}{2(1+\epsilon_3)} W p_{\text{sub}}} \Theta(1)
\]

as the throughput capacity corresponding to mobile ad hoc transmissions.

2) Average Packet Delay:

To compute the average delay of each packet, the first hitting time for a single node in a 2-d torus of size \( \sqrt{n} \times \sqrt{n} \) is pertaining.

Lemma 3: Let \( t^c \) denote the first hitting time for a single node on a 2-torus of size \( \sqrt{n} \times \sqrt{n} \), then \( E[t^c] = \Theta(n \log n) \), where \( E[t^c] \) is the expectation of \( t^c \).

In a given cell, if the destination/base station is not within the direct transmission range of source node, it transmits the packet to the relay node which in turn carries the packet to the intended destination/base station and transmits it. Let the time needed for the relay node to first hit the intended destination/base station be \( t^r \). Further, it is to be noted that each cell of size \( \frac{1}{\sqrt{m}} \times \frac{1}{\sqrt{m}} \) consists of \( \frac{n}{m} \) sub cells of area \( \frac{1}{m} \). Each, resulting in a discrete torus of size \( \sqrt{\frac{n}{m}} \times \sqrt{\frac{n}{m}} \). Then, by Lemma 3, the expectation of \( t^r \), denoted by \( E[t^r] \) is given as:

\[
E[t^r] = \Theta \left( \frac{n}{m} \log n \right)
\]

(2)

Let \( D(n, m) = E[H] + E[t^r] \) be the average packet delay, where \( E[t^r] \) is the mean number of hops taken by a packet for each S-D pair. Then,

\[
D(n, m) = \Theta(1) + \Omega \left( \frac{n}{m} \log n \right) > \Omega \left( \frac{n}{m} \log n \right)
\]

which is a lower bound on the expected average packet delay. As the base stations grow faster than \( n \) the number of hops taken by each packet for S-D pair is at most \( \Theta(1) \). As a result, the delay bound by \( \Theta(1) \) for \( m = \Omega(n) \) and by \( \Omega \left( \frac{n}{m} \log n \right) \) for \( m = O(n) \).

B. Upper bound: Capacity and Delay for mobile ad hoc transmissions

This section characterizes the upper bounds on the per-node throughput and delay of mobile ad hoc transmissions. The upper bound on throughput capacity is \( \frac{\lambda a(n,m)}{r(n)} = \frac{1}{r^2(n)} \), where \( \lambda a(n,m) \) is the number of hops taken by each packet. Thus, by replacing the factor \( \frac{1}{r(n)} \) by \( \Theta(1) \) (the reduced number of hops due to the mobility nature of the nodes) and setting the transmission range \( r^2(n) \) of each node to be \( 2\sqrt{2} \), we obtain \( \Lambdaa = O(Wa) \) as the upper bound on the throughput capacity. The upper bound on the average packet delay is \( D(n, m) \). For a 2-dimensional network of \( n \) nodes, the cover time is given as \( O(n \log n)[10] \). Thus, in MHWN model, since each node is limited to move around a cell consisting of \( n/m \) nodes, the cover time of a cell as \( O\left( \frac{n}{m} \log n \right) \) is obtained. This implies that the time needed for the relay node to first hit the intended destination/base station in a cell satisfies \( E[t^c] = O\left( \frac{n}{m} \log n \right) \). Besides, as the number of hops is \( \Theta(1) \), upper bound is \( D(n, m) \) as \( O\left( \frac{n}{m} \log n \right) \).

C. Capacity of base station relative transmissions

Lemma 4: Let be the bandwidth allocated for an uplink transmission and \( (n,m) \) be the number of nodes and base stations placed in a planar torus of unit area. Then, each node within a cell attains a throughput of \( \Lambdaa(n,m) = \Theta \left( \frac{m W}{n} \right) \) with some frequency reuse policy.

Proof: Since both the upper and lower bounds maps to each other, the tight bounds are denoted by \( \Theta(...) \) in Theorems 1 and 2 respectively. This concludes the proof. The capacity figures, \( \Lambdaa(n,m) \) and \( \Lambdaa(n, m) \), in Theorem 1 in fact depends on different channel allocation schemes. Therefore to get the maximum throughput capacity, one has to maximize the throughput over all possible combinations of \( W/\omega \). We then have the following cases:

I) when \( m = O(n) \), the maximum throughput capacity is achieved by allocating most of the bandwidth for mobile mode and only allocating a minimal amount of bandwidth for the base station relative transmissions in the hybrid mode. When \( W/\omega \rightarrow 0 \) or when \( W/\omega = W \), the throughput available for each node as \( \Lambda(n,m) = \Theta(W) \). From Section II-B, it follows that \( W = W + 2Wu \). Thus, by replacing \( W/\omega \) in Theorem 1 by \( W = W - 2Wu \) and rearranging some terms, \( \Lambda(n,m) = W + \left( 1 - 2 \frac{n}{m} \right) \frac{m}{n} Wu \).

Since the factor \( \left( 1 - 2 \frac{n}{m} \right) < 0 \) for \( m = O(n) \) the maximum Capacity has got by allocating most of the bandwidth for ad hoc mode transmissions i.e., \( W/\omega \rightarrow 1 \). (b) when \( m = \Omega(n) \), each node realizes the maximum throughput by allocating most of the bandwidth for carrying base station relative transmissions.
Alternatively, when $W_a / W \to 0$ or $W_u = W / 2$, the throughput has got available to each node as $\Lambda(n,m) = \Theta \left( \frac{W}{m} \right)$, when $m > n$, the number of nodes communicating in mobile mode within a cell decreases and hence, most of the traffic has to be carried through the base station which in turn requires larger bandwidth.

III. COMPARISON WITH EXISTING SOLUTIONS

In this section, mobile hybrid wireless network (MHWN) and static hybrid wireless network (SHWN) are designed with several existing schemes [9]. For the convenience of elucidation, the following terms are used for the pure ad hoc networks in [2], mobile ad hoc networks in [3] and $k$-nearest cell routing policy in [5] as “pure ad hoc network”, “MOBILITY”, and “k-nearest”, respectively.

To study the performance of design, two parameters are considered: capacity gain and delay gain denoted by $G_c$ and $G_d$ respectively;

Where $G_c = \frac{\text{Capacity of our Design}}{\text{Capacity of related Works}}$ and $G_d = \frac{\text{Delay of Related Works}}{\text{Delay of our Design}}$.

Essentially Figure 2 & Figure 3 compares the capacity and delay achieved under SHWN & MHWN respectively, with pure ad hoc networks and MOBILITY as $m$ increases. A better performance is observed for both SHWN and MHWN with $m$. A significant gain in capacity is realized by the MHWN in comparison to SHWN. This is due to the reduction of number of hops between each S-D pair because of the mobility nature and thereby the amount of relaying traffic per node.

IV. CONCLUSION

In this paper, two major questions are addressed:

(a) Can a hybrid wireless network design a solution with low deployment cost?

(b) Can a scheme design that provides $\Theta(W)$ throughput capacity, while possibly keeping end-to-end delay smaller? By exploiting the mobility characteristics of nodes in a hybrid network, it is shown that each node can be realized with a capacity of $\Theta(1)$, while keeping the average end-to-end delay smaller by a factor of $m$ than the traditional mobile ad hoc networks. Besides, in comparison to existing works, it is clearly shown that the gain obtained on delay as well as on capacity in implementing the design.

Figure 2 Network Capacity $n\Lambda(n,m)$ vs. $m$ for SHWN, $k$-nearest cell, MHWN.

Figure 3. Delay $D(n,m)$ vs. $m$ for SHWN, MHWN, Mobility and Pure Ad hoc Networks.

V. REFERENCES