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Plane Symmetric Dissipative Future Universe With-out Big Rip

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Abstract—Plane symmetric dissipative future universe without Big Rip in the context of Ecart formalism has been investigated. If cosmic dark energy behaves like a fluid with equation of state $p = \omega\rho$ (p and ρ being pressure and energy density respectively) as well as generalized Chaplygin gas simultaneously then Big Rip dose not arise even for equation of state parameter $\omega < -1$ and scale factor is found to be regular for all time.

Keywords—Phantom fluid, Big Rip, Plane symmetric universe.

I. INTRODUCTION

The recent cosmological observations of Type Ia Super Nova (SNe la) [1-8] states that our universe is in an accelerated expansion. This acceleration is due to exotic components with negative pressure called dark energy. It is understood that in the cosmological model, dark energy constitutes about 70% dark energy about 26% matter and baryon matter about 4%. Several dark energy cosmological models have been obtained by the cosmologist to explain dark energy. The equation of state for dark energy is given by $\omega = \frac{p}{\rho}$, where p is pressure and ρ is energy density [9]. Depending upon the value of ω, different types of dark energy models are obtained. When $\omega < -1$, dark energy model is and it is quintessence for $-1 < \omega < -\frac{1}{3}$. In the dark energy model when the parameter ω crosses the phantom divide line $\omega = -1$ both sides then it is termed as quintom.

In general, ω is a function of time. In Roberson Walker cosmology, the EOS for Chaplygin gas is given by

$$p = -\frac{K}{\rho}, \quad (1)$$

where p is pressure and ρ is energy density in reference frame with $\rho > 0$ and K is a positive constant.

The equation of state for generalized Chaplygin gas is given by

$$p = -\frac{K}{\rho^{\frac{1}{\alpha}}}, \quad (2)$$

with $1 \le \alpha < \infty$.

For $\alpha = 1$, equation (02) becomes (01).

Kamenshchilk*et al.* [10], Bento *et al.*, [11], Gorini*et al.*, [12], Bertolami*et al.* [13] have claimed that generalized Chaplygin gas (GCG) is better fit for latest supernova data.

A cosmological observation made in the late nineties and at the beginning of this century indicates the evidence for cosmic acceleration. It is driven by a fluid called as dark energy. Violating strong energy condition such that $\omega < -\frac{1}{3}$. The action obeying the phantom like behavior may be arise in super gravity [14], scalar tensor gravity [15], Higher derivative gravity [16], Brain world [17], String theory [18] and also from quantum effect[19-22]. The visible universe driven by the phantom field will evolved to a singularity in which the energy density becomes infinite at finite time which is called the Big Rip [23-27]. With the use of general relativity based on Friedman equations it is observed that phantom dominated epoch of the universe expands faster, but ends up in the form of Big Rip singularity in a finite future time [28]. Phantom dark energy fields are characterized by violating the main energy condition, $\rho + p > 0$. Also the conservation equation has the striking consequence that the energy density increases with expansion and the condition $\omega < -1$ matter is called phantom energy. Caldwell [6] noted that the EOS parameter ω has a very short range in the neighborhood of $\omega = -1$, with more likely hood to the side of $\omega < -1$.

In this paper, plane symmetric dissipative future universe without Big Rip has been investigated. This work is organized as follows: In section 2, the model and field equations have been presented. The field equations have been solved by using the physical condition that the expansion scalar θ is proportional to shear scalar σ, while concluding remarks have expressed in section 4.

II. METRIC AND FIELD EQUATIONS:

Plane Symmetric metric is given by

$$ds^2 = dt^2 - A^2(dx^2 + dy^2) - B^2 dz^2 . \quad (3)$$

The universe is assumed to be filled with distribution of matter represented by energy-momentum tensor for bulk viscous fluid given by

$$T_j^i = (\rho + p)u^i u_j - \bar{p} g_j^i , \quad (4)$$

with $\bar{p} = p - 3\xi H$, (5)

where ρ is an energy density, p is an isotropic pressure, $\bar{p}$ is an effective pressure, ξ is bulk viscous coefficient, H is Hubble's parameter, u^i is the four velocity of fluid which satisfy the condition

$$u^i u_j = 0 , \quad i = 1,2,3$$

and $u^0 u_0 = -1$.

Using the above equations; the matter tensor is given by

$$T_j^i = diag.(\rho, -p, -p, -p) . \quad (6)$$

The Einstein's field equations are

$$R_j^i - \frac{1}{2} g_j^i R = -T_j^i \,, \tag{7}$$

where R_j^i Ricci Tensor, R is is a Ricci scalar and T_j^i is an energy momentum tensor for bulk viscous cosmology.

Using equation (06), for metric (03), the Einstein field equations reduce to

$$2\frac{\dot{A}\dot{B}}{AB} + \frac{\ddot{A}}{A} = \rho \,, \tag{8}$$

$$\frac{\dot{A}^2}{A^2} + 2\frac{\ddot{A}}{A} = \bar{p} \,, \tag{9}$$

$$\frac{\ddot{A}}{A} + \frac{\dot{A}\dot{B}}{AB} + \frac{\ddot{B}}{B} = \bar{p} \,. \tag{10}$$

Here (.) Dot represents the differentiation with respect to t.

The energy conservation equation is given as

$$T^{ij}_{;j} = 0 \,, \tag{11}$$

where $T^{ij}_{;j} = \frac{1}{\sqrt{-g}} \frac{\partial}{\partial x^i} (T^{ij}\sqrt{-g}) + T^{jk}\Gamma^i_{jk}$ a .

Using equation (3), the above equation simplifies to

$$\dot{\rho} + (\rho + \bar{p})\left(2\frac{\dot{A}}{A} + \frac{\dot{B}}{B}\right) = 0 \,, \tag{12}$$

where dot is the differentiation with respect to t.

III. SOLUTION OF THE FIELD EQUATIONS:

In order to obtain the solution, we have three linearly independent equations [8-11] and five unknowns A, B, ρ, p and ξ. To get deterministic solution, we need two additional conditions. First we use the physical condition that expansion scalar θ is proportional to shear scalar σ, which leads to

$$B = A^m \,, \tag{13}$$

where $m(>0)$ is a constant. [29]

With the help of equation (13), equation (11) reduces to

$$\frac{\dot{A}^2}{A^2} = \frac{\rho}{2m+1} \,. \tag{14}$$

From equation (13), we get

$$3H = (m+2)\frac{\dot{A}}{A} \,. \tag{15}$$

Using equations (13), (14), and (15), equation (12) leads to

$$\dot{\rho} + 3H(\bar{p} + \rho) = 0 \,. \tag{16}$$

Using equation (3), equation (16) can be written as

which simplifies to

$$\dot{\rho} + 3H\left[\frac{\rho^{\frac{1+\alpha}{\alpha}} - K}{\rho^{1/\alpha}}\right] - 3\xi H = 0 \,. \tag{17}$$

To obtain the solution of (17), we assume the viscosity has a power law dependence upon the density given by

$$\xi = \xi_0 \rho^n \,, \tag{18}$$

where ξ_0 and n are constants.

From equations (17) and (18) we get

$$\frac{d\rho}{dt} + 3H\left[\frac{\rho^{\frac{1+\alpha}{\alpha}} - K}{\rho^{1/\alpha}}\right] = \frac{(m+2)^2}{(2m+1)} \xi_o \rho^{n+1} \tag{19}$$

To solve (19), use the transformation $\frac{d\rho}{dt} = \dot{\rho} = \rho^{n+1}$ in equation (19) which reduces to

$$\rho^{\frac{1+\alpha}{\alpha}} = K + \left[\rho_0^{\frac{1+\alpha}{\alpha}} - K\right]\left(\frac{A_0}{A}\right)^{\left[\frac{(2m+1)(m+2)}{(2m+1)-(m+2)^2\xi_0}\right]\left[\frac{1+\alpha}{\alpha}\right]}, \tag{20}$$

where ρ_0, A_0 represent the value of $\rho(t)$ and $A(t)$ at present time t_0 respectively.

It is assumed that dark energy behaves like GCG obeying equation (2) with $1 \le \alpha < \infty$ 1, as well as fluid with equation of state

$$p = \omega\rho \,, \tag{21}$$

with $\omega < -1$ simultaneously.

From equations (2) and (21) we get

$$\omega(t) = -\frac{K}{\rho^{\frac{1+\alpha}{\alpha}}} \,. \tag{22}$$

At $t = t_0$ equation (22) gives

$$K = -\omega_0 \rho_0^{\frac{1+\alpha}{\alpha}} \,, \tag{23}$$

whereω_0 is value of $\omega = \omega(t)$ at $t = t_0$.

With the help of equation (23), equation (20) leads to

$$\rho = \rho_0\left[\left[-\omega_0 + (1+\omega_0)\right]\left(\frac{A_0}{A}\right)^{\left[\frac{(2m+1)(m+2)}{(2m+1)-(m+2)^2\xi_0}\right]\left[\frac{1+\alpha}{\alpha}\right]}\right]^{\frac{\alpha}{1+\alpha}} \tag{24}$$

In homogeneous model of universe, scalar fields $\phi(t)$ with potential $V(\phi)$ has energy density ρ_ϕ and pressure p_ϕ respectively are

$$\rho_\phi = \frac{1}{2}\dot{\phi}^2 + V(\phi) \tag{25}$$

and $$p_\phi = \frac{1}{2}\dot{\phi}^2 - V(\phi) \tag{26}$$

where $\dot{\phi}$ is differentiation of ϕ with respect to t.

Adding equations (25) and (26) we get $\dot{\rho} + 3H\left[\rho + p - 3\xi H\right] = 0$.

$$\dot{\phi}^2 = \rho_\phi + p_\phi \,. \tag{27}$$

With the help of equations (2) and (23), equation (27) reduces to

$$\dot{\phi}^2 = \frac{\rho^{\frac{1+\alpha}{\alpha}} + \rho_0^{\frac{1+\alpha}{\alpha}}\omega_0}{\rho^{\frac{1}{\alpha}}} \,. \tag{28}$$

With the help of equation (24), equation (28) leads to

$$\dot{\phi}^2=\frac{(1+\omega_0)\rho_0\left(\frac{A_0}{A}\right)^{\left[\frac{(m+2)(2m+1)}{(2m+1)-(m+2)^2\xi_0}\right]\left[\frac{1+\alpha}{\alpha}\right]}}{\left[-\omega_0+(1+\omega_0)\left(\frac{A_0}{A}\right)^{\left[\frac{(m+2)(2m+1)}{(2m+1)-(m+2)^2\xi_0}\right]\left[\frac{1+\alpha}{\alpha}\right]}\right]^{\frac{\alpha}{1+\alpha}}}$$

.

From equation (27), it is observed that when $\dot{\phi}^2>0$ we get positive kinetic energy for $(1+\omega_0)>0$ and when $\dot{\phi}^2<0$ we get negative kinetic energy $(1+\omega_0)<0$. Thus $(1+\omega_0)>0$ represents a case of quintessence and $(1+\omega_0)<0$ represent phantom fluid dominated universe which matches with the results obtained by Hoyle and Narlikar in C- field with negative kinetic energy for steady state theory of universe.

Now from equations (14) and (24), we get

$$\left(\frac{\dot{A}^2}{A^2}\right)^2=\left(\frac{3}{2m+1}\right)\Omega_0H_0^2$$

$$\left[|\omega_0|+(1-|\omega_0|)\left(\frac{A_0}{A}\right)^{\left[\frac{(m+2)(2m+1)}{(2m+1)-(m+2)^2\xi_0}\right]\left[\frac{1+\alpha}{\alpha}\right]}\right]^{\frac{\alpha}{1+\alpha}},$$

(30)

where $|\omega_0|=-\omega_0$, $H_0=100\,h\,km/s\,mpc$ present value of Hubble parameter and $\Omega_0=\frac{\rho_0}{\rho_{cr,o}}$ with $\rho_{cr,o}=\frac{3H_0^2}{8\pi G}$.

Taking square root to both sides of equation (30) gives

$$\frac{\dot{A}}{A}=\sqrt{\frac{3\Omega_0}{2m+1}}H_0|\omega_0|^{\frac{\alpha}{2(1+\alpha)}}$$

$$\left[1+\frac{(1-|\omega_0|)}{|\omega_0|}\left(\frac{A_0}{A}\right)^{\left[\frac{(m+2)(2m+1)}{(2m+1)-(m+2)^2\xi_0}\right]\left[\frac{1+\alpha}{\alpha}\right]}\right]^{\frac{\alpha}{2(1+\alpha)}}$$

(31)

Expanding the R.H.S. of (31) and neglecting the higher powers of $\left[1+\frac{(1-|\omega_0|)}{|\omega_0|}\left(\frac{A_0}{A}\right)^{\left[\frac{(m+2)(2m+1)}{(2m+1)-(m+2)^2\xi_0}\right]\left[\frac{1+\alpha}{\alpha}\right]}\right]$ we get

$$\frac{\dot{A}}{A}=\sqrt{\frac{3\Omega_0}{2m+1}}H_0|\omega_0|^{\frac{\alpha}{2(1+\alpha)}}$$

$$\left[1+\frac{\alpha(1-|\omega_0|)}{2(1+\alpha)|\omega_0|}\left(\frac{A_0}{A}\right)^{\left[\frac{(m+2)(2m+1)}{(2m+1)-(m+2)^2\xi_0}\right]\left[\frac{1+\alpha}{\alpha}\right]}\right]. \qquad (32)$$

On Integration of equation (32)

$$A(t)=\frac{A_0}{\left[2(1+\alpha)|\omega_0|\right]^{\left[\frac{(m+2)(2m+1)}{(2m+1)-(m+2)^2\xi_0}\right]\left[\frac{1+\alpha}{\alpha}\right]}}$$

$$\times\left[\alpha+2(1+\alpha)|\omega_0|\left[\frac{(m+2)(2m+1)}{(2m+1)-(m+2)^2\xi_0}\right]\left[\frac{1+\alpha}{\alpha}\right]\sqrt{\frac{3\Omega_0}{2m+1}}-\alpha(1-|\omega_0|)\right]^{\left[\frac{(m+2)(2m+1)}{(2m+1)-(m+2)^2\xi_0}\right]\left[\frac{1+\alpha}{\alpha}\right]}$$

(29)

(33)

From equation (33), it is clear that as $t\to\infty$, $A(t)\to\infty$ therefore the present model is free from finite time future singularity.

In this case, the Hubble distance is given by

$$H^{-1}=\frac{\sqrt{\frac{3(2m+1)}{(m+2)^2}}}{\sqrt{\Omega_0}\,H_0|\omega_0|^{\frac{\alpha}{2(1+\alpha)}}}$$

$$\left[1-\frac{\alpha(1-|\omega_0|)}{2(1+\alpha)|\omega_0|}\left(\frac{A_0}{A}\right)^{\left[\frac{(m+2)(2m+1)}{(2m+1)-(m+2)^2\xi_0}\right]\left[\frac{1+\alpha}{\alpha}\right]}\right]$$

(34)

Equation (34) shows the growth of Hubble distance H^{-1} with time such that

$$H^{-1}\to\frac{\sqrt{\frac{3(2m+1)}{(m+2)^2}}}{\sqrt{\Omega_0}\,H_0|\omega_0|^{\frac{\alpha}{2(1+\alpha)}}}\neq 0 \text{ as } t\to\infty .$$

Thus in present plane symmetric universe, the galaxies will not disappear as $t\to\infty$, avoiding Big Rip singularity. Therefore one can conclude that if Phantom fluid behaves like GCG and fluid with $p=\omega\rho$ simultaneously then the future accelerated expansion of the universe will free from catastrophic situation like Big Rip in plane symmetric universe.

Equation (24) can be written as

$$\rho=\rho_0\left[\left[|\omega_0|+(1-|\omega_0|)\left(\frac{B_0}{B}\right)^{\frac{(2m+1)(m+2)(1+\alpha)}{\alpha\left[(2m+1)-(m+2)^2\xi_0\right]}}\right]^{\frac{\alpha}{1+\alpha}}\right.$$

(35)

From equation (35), it is clear that as $t\to\infty$,

$\rho\to\rho_0|\omega_0|^{\frac{\alpha}{1+\alpha}}>\rho_0$.

Thus one can conclude that energy density increases with time, contrary to other Phantom models having future singularity at $t=t_s$ in plane symmetric universe.

IV. Conclusion

Plane symmetric universe is studied when cosmic dark energy behaves simultaneously like a fluid with equation of state $p=\omega\rho$, $\omega<-1$ as well as generalized Chaplygin gas with equation of state $p=-\frac{K}{\rho^{1/\alpha}}$.

The concluding remarks are as follows:

i) $\dot{\phi}^2 > 0$ we get positive kinetic energy when $(1+\omega_0) > 0$ re-presenting the case of quintessence and for $\dot{\phi}^2 < 0$, we get negative kinetic energy $(1+\omega_0) < 0$ when representing phantom field dominated universe. These results match with the results obtained by Hoyle-Narlikar in C-field with negative kinetic energy for steady state of universe.

ii) As $t \to \infty$, $A(t) \to \infty$ indicating that the model is free from finite future singularity.

iii) As $t \to \infty$, H^{-1} indicating that at present time the galaxies will not disappear as $t \to \infty$, avoiding Big Rip singularities.

iv) As $t \to \infty$, $\rho \to \rho_0 |\omega_0|^{\frac{\alpha}{1+\alpha}} > \rho_0$ concluding that energy density increases with time contrary to other phantom models having future singularity at $t = t_s$.

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